

Analytical Determination of Electric Voltage for Pressure-driven Flow Through Complex Microchannels

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Abstract

An analytical determination of electric voltage generated by the pressure-driven flow through complex microchannels was analyzed based on the fractal theory. The pressure-driven flow through a complex microchannel with consideration of electrokinetic phenomena is described by the momentum and Poisson-Boltzmann (P-B) equations. The solution of induced electric field strength and electric voltage across complex microchannels are obtained using the fractal theory and technique, which are the function of dimensionless electroosmotic radius and the porosity. The results obtained show that the analytical results are agreed well with the experimental data.

Keywords: Fractal Theory; Electrical Double Layer; Electric Voltage; Porosity

1 Introduction

Complex microchannels are widely used in several engineering areas including thermal insulation, heat exchangers, filtration and separation of particles, composite fabrication, and medical science. Most solid surfaces carry electrostatic charges which produces an electrical surface potential. If the liquid contains a very small number of ions in the liquid. The rearrangement of the charges on the solid surface and the balancing charges in the liquid is called the Electrical Double Layer (EDL). It has been proved that the flows in micro-scale are quite different from those in macro-scale, suggesting that the electrokinetic phenomena caused by EDL effects must be taken into account in micro-channel flows [1-5].

The disordered nature of pore structure in complex microchannels shows the use of a fractal structure formed by both micro-pores inside the porous fibrous materials [6]. Applications of the fractal theory to analyze transport properties of complex microchannels in science and engineering have received increasing attention in the past two decades [7, 8]. A fractal geometry model for evaluating permeability of porous textile is studied, which is used in liquid composite molding [9]. A mathematical model for the coupled heat and mass transfer in complex microchannels is established based on the fractal characters of the pore size distribution [10].

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The notion of generating electrical power in this way is an old topic, yet has received renewed attention in the context of the voltage generation by pressure-driven through complex microchannels. Pressure-driven flow through a microchannel induces a flow of charges in the double layer at the channel walls, generating an electrical current. The efficiency of electrical power generation is discussed in individual rectangular nano-channels by means of streaming currents and the pressure-driven transport of counter ions in the EDL [11]. A mathematical model is presented to describe resistance effects of the EDL in porous textiles on the coupled heat and liquid moisture transfer [12]. The fractal dimensions and the micro-structural parameters of fractal complex microchannels are analyzed to study the mass transfer for fluid flow [13].

There were rarely works analyzing the electric voltage generated by pressure-driven flow through complex microchannels on the basis of the fractal analysis of pore microstructures. Taking into account the electrokinetic effect, the momentum equation describing the pressure-driven flow as well as the P-B description of induced electric potential distributions is presented. The fractal model of the electric voltage across complex microchannels generated by pressure-driven flow is derived based on the fractal characters of complex microchannels. It is found that the electric voltage is a function of porosity, the dimensionless local averaging net charge density, the dimensionless electroosmotic radius, length of the complex microchannel, the pressure across the complex microchannel, the solid surface zeta potential and so on. Furthermore, the predicted model for voltage shows good agreements with experimental data, illustrating that the analytical determination for the electric voltage across the complex microchannels is satisfactory, which can be applied to measure the actual voltage across complex microchannels and adjust the magnitude through changing the concerning parameters in microfluidic system.

2 Analysis of Mathematical Model

Firstly we considered the steady flow of an electrolyte solution through a single microchannel. Based on the P-B equation, the electric potential ψ can be expressed as [6]:

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\psi}{dr} \right) = -\frac{\rho_e}{\varepsilon} \quad (1)$$

The net charge density is given as [14]:

$$\rho_e = -2\chi en_0 \sinh \left(\frac{\chi e \psi}{k_b T} \right) \quad (2)$$

where n_0 and χ are the bulk ionic concentration and the valence of ions, respectively, ε is the permittivity or the dielectric constant, e is the charge of an electron, k_b is the Boltzmann constant and T is the absolute temperature. By using following non-dimensional quantities $\bar{\psi} = \frac{\psi}{\zeta}$, $\bar{r} = \frac{r}{r_c}$, $\bar{\rho}_e = \frac{\rho_e}{-\varepsilon \zeta / r_c^2}$, where ζ denotes the Zeta potential on the surfaces of channelwall and r_c is the pore radius of single microchannel, $k = \kappa r_c$ and k is the dimensionless electroosmotic radius, where $\kappa = \sqrt{2n_0 e^2 \chi^2 / \varepsilon k_b T}$.

Eq. (1) can be expressed as:

$$\frac{1}{\bar{r}} \frac{d}{d\bar{r}} \left(\bar{r} \frac{d\bar{\psi}}{d\bar{r}} \right) = k^2 \sinh(\bar{\psi}) \quad (3)$$

The corresponding boundary conditions are:

$$\bar{r} = 0, \quad \frac{d\bar{\psi}}{d\bar{r}} = 0; \quad \bar{r} = 1, \quad \bar{\psi} = 1 \quad (4)$$

It is assumed that the magnitude of zeta potential is very small, i.e. $\zeta \leq 25 \text{ mV}$, meaning physically that the electrical potential is small compared with the thermal energy of the charged species, with the result the electric potential due to charged wall can be described by linearizing hyperbolic sine function $\sinh(x)$ with x when x is small. It is also called Debye-Huckel linearization approximation [12]. Then Eq. (3) can be reduced to:

$$\frac{1}{\bar{r}} \frac{d}{d\bar{r}} \left(\bar{r} \frac{d\bar{\psi}}{d\bar{r}} \right) = k^2 \bar{\psi} \quad (5)$$

The solution of Eq. (5) is $\bar{\psi} = \frac{I_0(k\bar{r})}{I_0(k)}$, where I_0 is the zero-order modified Bessel function, therefore, Eq. (2) is written as the dimensionless form:

$$\bar{\rho}_e = \frac{1}{\bar{r}} \frac{d}{d\bar{r}} \left(\bar{r} \frac{d\bar{\psi}}{d\bar{r}} \right) = k^2 \bar{\psi} = k^2 \frac{I_0(k\bar{r})}{I_0(k)} \quad (6)$$

where $\bar{\rho}_e = \frac{\rho_e}{-\varepsilon\zeta/r_c^2}$ denotes the dimensionless net charge density. The momentum equation describing the pressure-driven flow taking into account the electrical body force can be given as:

$$\mu \frac{1}{r} \frac{d}{dr} \left(r \frac{du}{dr} \right) = p_z - \rho_e E \quad (7)$$

where μ is the viscosity of fluid, u is the velocity of fluid, which is the function of r only, ρ_e is the local net charge density per unit volume. When the ions move in the solution, $\rho_e E$ presents the electro-viscosity term. They exert a force on the liquid molecules, thus generating a viscous effect, which is called the electrokinetic effect.

$$J_s = u(r) \rho_e(r) \quad (8)$$

where J_s is the electric current density. The stream electric field can be obtained through a balance condition between streaming current and electrical conductance current at steady state. In the steady flow we have $J_s + J_c = 0$, and

$$E(r_c) = \frac{J_c(r)}{\lambda} \quad (9)$$

where λ is the electric conductivity of the solution referring to the total electrical conductivity. According to Eqs. (7) and (9), the following equation is obtained:

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{du}{dr} \right) - \frac{\rho_e^2 u}{\mu \lambda} = \frac{1}{\mu} p_z \quad (10)$$

and the solution [12] is

$$u(r) \approx \frac{p_z \lambda}{4\mu \lambda + \rho_e^2 r_c^2} (r^2 - r_c^2) = \frac{p_z \lambda r_c^2}{4\mu \lambda r_c^2 + \bar{\rho}_e^2 \varepsilon^2 \zeta^2} (r^2 - r_c^2), \quad (r \leq r_c) \quad (11)$$

According to Eqs. (8), (9), and $J_s + J_c = 0$,

$$E(r) = -\frac{\bar{\rho}_e p_z \varepsilon \zeta}{4\mu\lambda r_c^2 + \bar{\rho}_e^2 \varepsilon^2 \zeta^2} (r^2 - r_c^2), \quad (r \leq r_c) \quad (12)$$

$$E^*(r) = -\frac{\bar{\rho}_e p_z \varepsilon \zeta}{4\mu\lambda r_c^2 + \bar{\rho}_e^2 \varepsilon^2 \zeta^2} (r^2 - r_c^2), \quad (r \leq r_c) \quad (13)$$

where $\bar{\rho}_e = \frac{2\pi \int_0^1 \bar{\rho}_e \bar{r} d\bar{r}}{\pi \times 1^2} = 2 \int_0^1 \bar{\rho}_e \bar{r} d\bar{r}$. Without consideration of the electrokinetic effect in the steady flow, the electric field strength \tilde{E} is obtained from the Helmholtz-Smoluehowski equations

$$\tilde{E}(r) = -\frac{p_z \varepsilon \zeta}{\mu\lambda} \quad (14)$$

Eqs. (12) and (13) are non-dimensionalized and we get:

$$s1 = \frac{E(r)}{\tilde{E}} = \frac{\frac{\bar{\rho}_e p_z \varepsilon \zeta}{4\mu\lambda r_c^2 + \bar{\rho}_e^2 \varepsilon^2 \zeta^2} (r^2 - r_c^2)}{\frac{p_z \varepsilon \zeta}{\mu\lambda}} = \frac{\bar{\rho}_e}{4 + \bar{\rho}_e^2 \Theta} (\bar{r}^2 - 1) \quad (15)$$

$$s2 = \frac{E^*(r)}{\tilde{E}} = \frac{\frac{\bar{\rho}_e p_z \varepsilon \zeta}{4\mu\lambda r_c^2 + \bar{\rho}_e^2 \varepsilon^2 \zeta^2} (r^2 - r_c^2)}{\frac{p_z \varepsilon \zeta}{\mu\lambda}} = \frac{\bar{\rho}_e}{4 + \bar{\rho}_e^2 \Theta} (\bar{r}^2 - 1) \quad (16)$$

It is shown in Fig. 1 that the proposed analytical determination of electric field strength represented by curves $s1$ has good agreements with the determination presented by $s2$ under the situations of $k = 5$, $k = 10$ and $k = 15$. The difference between the two curves is very small in the case of $k = 20$, where the error can be neglected. The fact that $E(r)$ is replaced by $E^*(r)$ is acceptable. Therefore, the method proposed is used to simplify the following derivation of electric field strength and electric voltage accurately. The mean electric field $\bar{E}(r_c)$ in single microchannel with the radius r_c can be deduced as follows:

$$\bar{E}(r_c) = \frac{\int_0^{r_c} E^*(r) d(\pi r^2)}{\pi r_c^2} = -\frac{2p_z \varepsilon \zeta}{\mu\lambda} \int_0^1 \frac{\bar{\rho}_e}{4 + \frac{\bar{\rho}_e^2 \varepsilon^2 \zeta^2}{\mu\lambda r_c^2}} (\bar{r}^3 - \bar{r}) d\bar{r} \quad (17)$$

We define $\Theta = \frac{\varepsilon^2 \zeta^2}{\mu\lambda r_c^2}$, Eq. (17) can be rewritten as:

$$\bar{E}(r_c) = -\frac{2p_z \varepsilon \zeta}{\mu\lambda} \int_0^1 \frac{\bar{\rho}_e}{4 + \bar{\rho}_e^2 \Theta} (\bar{r}^3 - \bar{r}) d\bar{r} \quad (18)$$

Substituting Eq. (6) into Eq. (18), it is obtained:

$$\bar{E}(r_c) = -\frac{2p_z \varepsilon \zeta}{k^2 \mu\lambda I_0(k) (4 + \bar{\rho}_e^2 \Theta)} \int_0^k I_0(x) (x^3 - k^2 x) d(x) \quad (19)$$

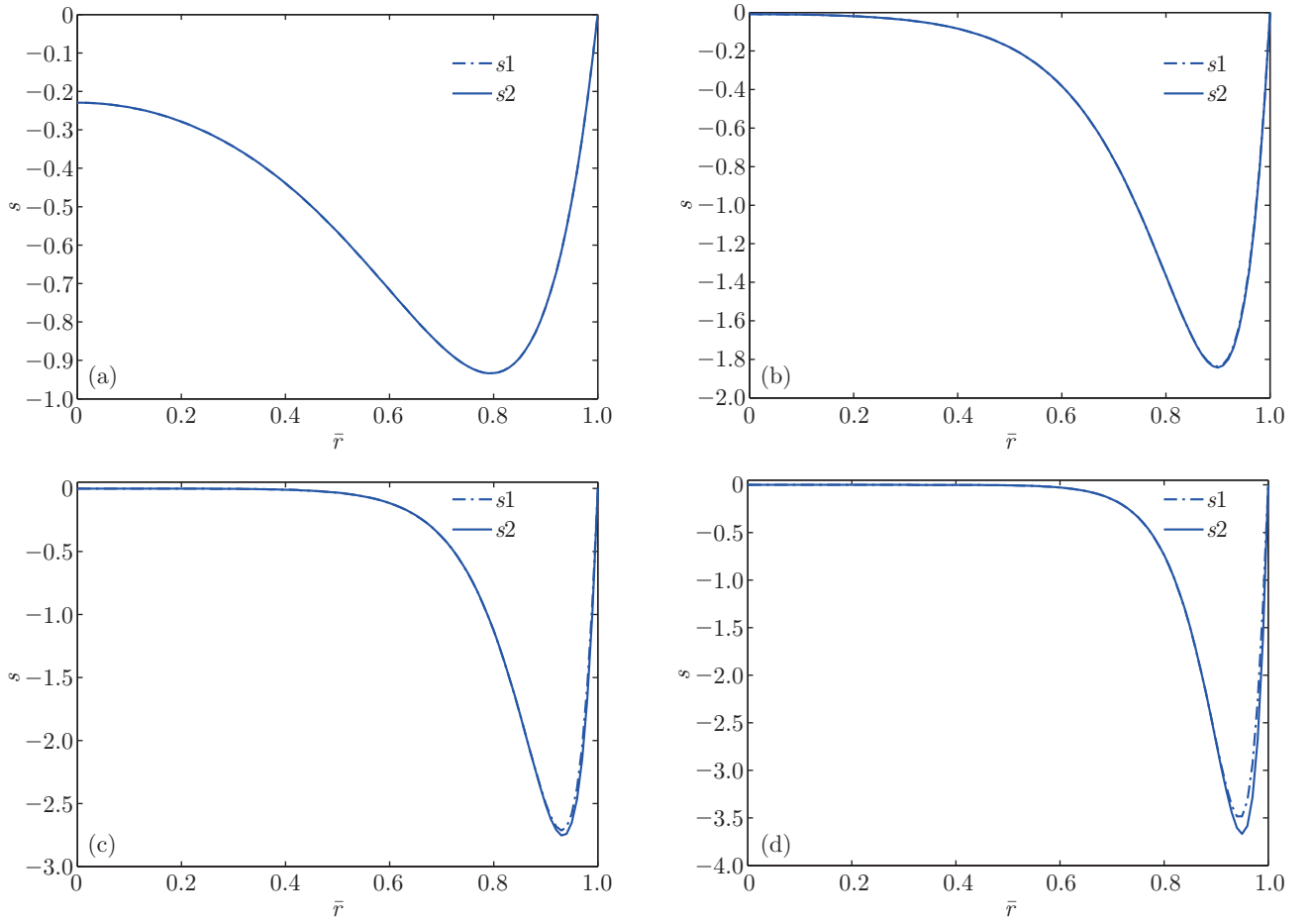


Fig. 1: Distributions of the dimensionless electric field strength (a) $k=5$, (b) $k=10$, (c) $k=15$ and (d) $k=20$

According to the expression of the zero-order modified Bessel function, Eq. (19) can be solved as:

$$\bar{E}(r_c) = \frac{4p_z \varepsilon \zeta}{\mu \lambda I_0(k) \left(4 + \frac{\varepsilon^2}{\rho_e^2} \Theta\right)} \sum_{i=0}^{\infty} \frac{1}{i! (i+2)!} \frac{x^{2i+2}}{2^{2i+2}} \quad (20)$$

with the expression of the second-order modified Bessel function, Eq. (20) is rewritten as:

$$\bar{E}(r_c) = \frac{4p_z \varepsilon \zeta I_2(k) r_c^2}{I_0(k) \left(4\mu \lambda r_c^2 + \frac{\varepsilon^2}{\rho_e^2} \varepsilon^2 \zeta^2\right)} \quad (21)$$

where I_0 and I_2 are the zero-order and the second-order modified Bessel function, respectively.

Due to the complicated characteristics of complex microchannels, the structure is considered to satisfy the fractal theory and technique. Therefore, based on fractal characters of complex microchannels and the probability theory [15], the traditional method of variable transformation is employed to solve the electric flux, which refers to the flow rate of the electric field through the area of total pores:

$$E(flux) = \int_{r_{\min}}^{r_{\max}} \bar{E}(r_c) f(r_c) N_t dr_c \quad (22)$$

where $f(r_c)$ is the probability density function of the pore distribution, N_t is the number of pores from the minimum radius r_{min} to the maximum radius r_{max} .

Substituting Eq. (21) into Eq. (22), it is obtained:

$$E(flux) = \int_{r_{min}}^{r_{max}} \frac{4p_z \varepsilon \zeta I_2(k) r_c^2}{I_0(k) \left(4\mu\lambda r_c^2 + \frac{\varepsilon^2}{\rho_e^2} \zeta^2\right)} D_F r_{max}^{D_F} r_c^{-1-D_F} dr_c \quad (23)$$

with the replacement of $x = kr_c$, and Eq. (23) can be transformed into:

$$E(flux) = \frac{p_z \varepsilon \zeta D_F r_{max}^{D_F} \kappa^{D_F}}{\mu\lambda} \int_{\kappa r_{min}}^{\kappa r_{max}} \left(\left(1 - \frac{2I_1(x)}{xI_0(x)}\right) \frac{x^{1-D_F}}{x^2 + \frac{\varepsilon^2}{\rho_e^2} \zeta^2 \kappa^2} \right) dx \quad (24)$$

Using the characters of Bessel functions, we get:

$$E(flux) = \frac{p_z \varepsilon \zeta D_F r_{max}^{D_F} \kappa^{D_F}}{3\mu\lambda} \int_{\kappa r_{min}}^{\kappa r_{max}} \left(\frac{x^{1-D_F}}{x^2 + \frac{\varepsilon^2}{\rho_e^2} \zeta^2 \kappa^2} \right) dx \quad (25)$$

by setting $t = x^2$, Eq. (25) is transformed into:

$$E(flux) = \frac{p_z \varepsilon \zeta D_F r_{max}^{D_F} \kappa^{D_F}}{6\mu\lambda} \frac{4\mu\lambda}{\frac{\varepsilon^2}{\rho_e^2} \zeta^2 \kappa^2} \int_{(\kappa r_{min})^2}^{(\kappa r_{max})^2} \left(\frac{t^{-\frac{D_F}{2}}}{1 + \frac{4\mu\lambda}{\frac{\varepsilon^2}{\rho_e^2} \zeta^2 \kappa^2} t} \right) dt \quad (26)$$

Using the integration by parts, we get

$$\begin{aligned} E(flux) = & \frac{p_z \varepsilon \zeta D_F r_{max}^{D_F} \kappa^{D_F}}{6\mu\lambda} \left[t^{-\frac{D_F}{2}} \ln \left(1 + \frac{4\mu\lambda}{\frac{\varepsilon^2}{\rho_e^2} \zeta^2 \kappa^2} t \right) \right]_{(\kappa r_{min})^2}^{(\kappa r_{max})^2} - \\ & \frac{D_F}{2} \left(\frac{4\mu\lambda}{\frac{\varepsilon^2}{\rho_e^2} \zeta^2 \kappa^2} \right)^{\frac{D_F}{2}} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n \left(n - \frac{D_F}{2} \right)} \left(\frac{4\mu\lambda}{\frac{\varepsilon^2}{\rho_e^2} \zeta^2 \kappa^2} t \right)^{n-\frac{D_F}{2}} \Bigg|_{(\kappa r_{min})^2}^{(\kappa r_{max})^2} \end{aligned} \quad (27)$$

and it is simply calculated:

$$E(flux) = \frac{p_z \varepsilon \zeta D_F}{6\mu\lambda} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n - \frac{D_F}{2}} \left(\frac{4\mu\lambda r_{max}^2}{\frac{\varepsilon^2}{\rho_e^2} \zeta^2} \right)^n (1 - \delta^{2n-D_F}) \quad (28)$$

where $D_F = d - \frac{\ln \phi}{\ln \delta}$, D_F being the fractal dimension of the object, and d , ϕ and δ are the Euclidean dimension, the porosity and $\delta = r_{min}/r_{max}$, respectively.

Hence, the electric voltage across the complex microchannels with length of L is

$$U = \frac{E(flux)L}{A_c} = \frac{Lp_z\varepsilon\zeta D_F}{6\mu\lambda A_c} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n - \frac{D_F}{2}} \left(\frac{4\mu\lambda r_{\max}^2}{\overline{\rho_e}^2 \varepsilon^2 \zeta^2} \right)^n (1 - \delta^{2n-D_F}) \quad (29)$$

in which A_c is the area of total pores [6] and $A_c = \frac{\pi D_F}{2 - D_F} r_{\max}^2 (1 - \delta^{2-D_F})$.

The relation between the macropore size in term of the porosity and the average radius of the complex microchannels R is as follows [16]:

$$r_{\max} = \frac{R}{4} \left(\sqrt{\frac{2\phi}{1-\phi}} + \sqrt{\frac{2\pi}{\sqrt{3}(1-\phi)}} - 2 \right) \quad (30)$$

Substituting Eq. (30) into Eq. (29), it is obtained:

$$U = \frac{8Lp_z\varepsilon\zeta(2-D_F)}{3\pi\mu\lambda R^2(1-\delta^{2-D_F})} \left(\sqrt{\frac{2\phi}{1-\phi}} + \sqrt{\frac{2\pi}{\sqrt{3}(1-\phi)}} - 2 \right)^{-2} \times \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(1-\delta^{2n-D_F})}{n - D_F/2} \left(\frac{\mu\lambda R^2}{4\overline{\rho_e}^2 \varepsilon^2 \zeta^2} \left(\sqrt{\frac{2\phi}{1-\phi}} + \sqrt{\frac{2\pi}{\sqrt{3}(1-\phi)}} - 2 \right)^2 \right)^n \quad (31)$$

3 Results and Discussion

We consider the KCl solution driven through the complex microchannels, and other parameters are taken as follows: absolute temperature $T = 298.15K$, $L = 4.5 \times 10^{-3}m$, $\phi = 0.475$, the dielectric constant $\varepsilon = 7.0832 \times 10^{-10}F/m$. In the following calculations, $\zeta = -25mV$ that is suitable for the linear approximation, $\mu = 0.8973 \times 10^{-3}Pa \cdot s$, the electric conductivity of the solution $\lambda = 1.288 \times 10^{-4}S/m$.

Fig. 2 (a) shows the comparison of the voltage versus pressure between analytical and experimental data. In the case that the concentration of KCl solution is taken, $1.0 \times 10^{-3}mol/L$, the voltage across the complex microchannels is computed by Eq. (31), and the results are consistent with experimental data [11]. Fig. 2 (b) shows a comparison of the analytical results and the measured results [17]. The results match extremely well with experimental data from the literatures. It turns out that the analytical determination of electric voltage across complex microchannels induced by pressure-driven flow is feasible.

Fig. 3 provides a detailed insight into the variation of the electric voltage with the porosity of complex microchannels for different dimensionless electroosmotic radius and radius ratio δ . It can be seen that the electric voltage decreases with the increase of the dimensionless electro-osmotic radius for a fixed δ . Increasing the value of k indicates that the density of free charged ions decreases, leading to the reduction of the voltage. The value of voltage is proportional to the porosity. The linear relationship is shown between the voltage and the porosity when the value of porosity increases. The comparison provided in Fig. 3, indicates that the dimensionless parameter δ has little influence on the electric voltage across the complex microchannels.

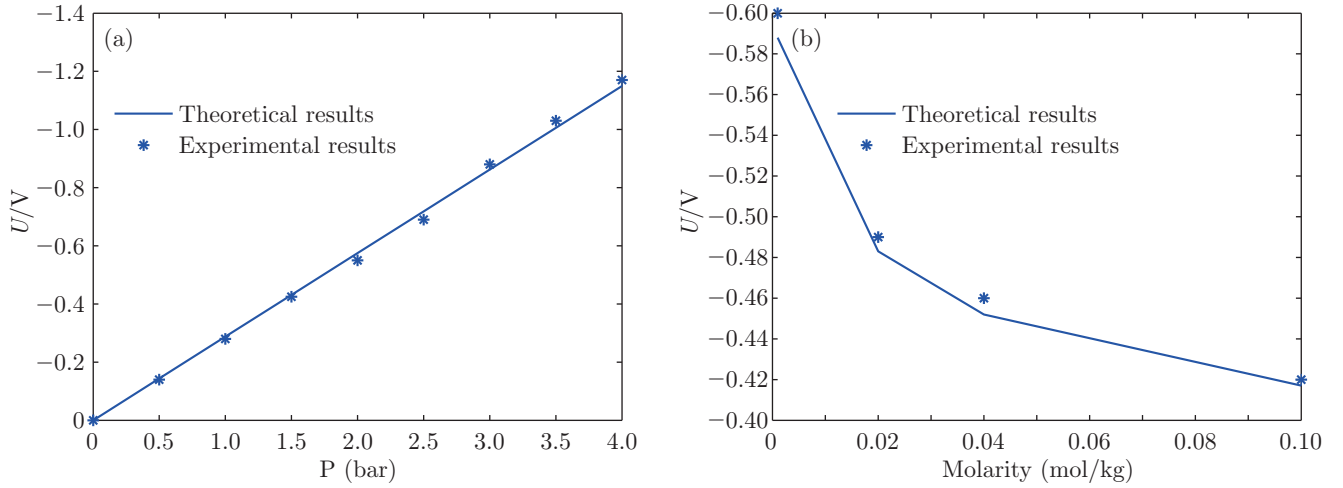


Fig. 2: (a) Comparison of the voltage versus pressure between analytical results and experiments [11], (b) Comparison of the voltage versus concentration between analytical results and experiments [17]

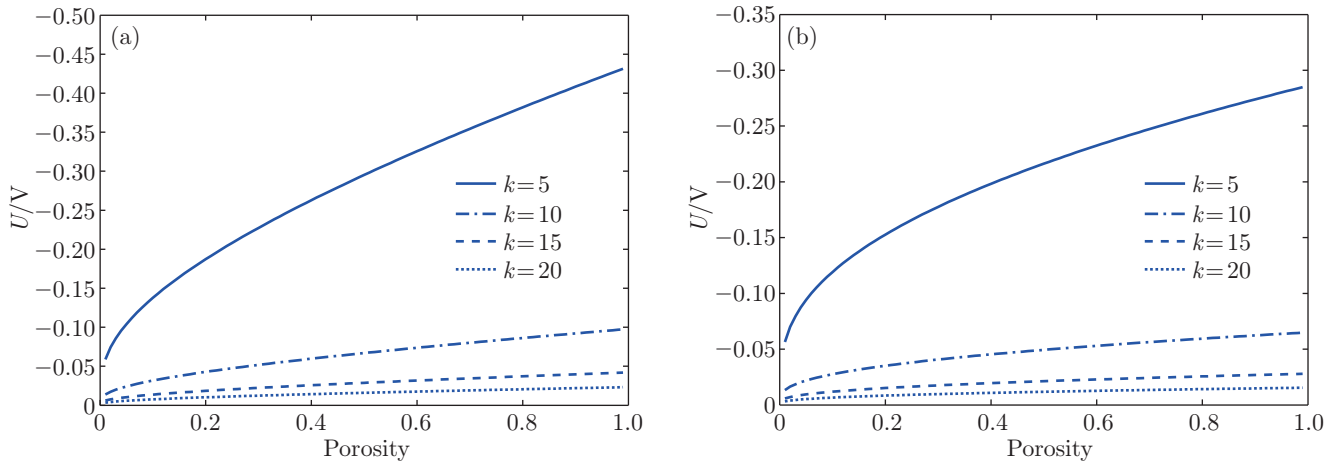


Fig. 3: Comparison of the voltage versus the porosity between different dimensionless electroosmotic radius (a) $\delta = 0.001$, (b) $\delta = 0.01$

4 Conclusions

The contribution of this paper is that an analytical determination for the electric voltage across complex microchannels generated by pressure-driven flow is firstly solved on the basis of the fractal theory and technique. The comparison between the predictions of the voltage and the related experimental data shows good agreements, demonstrating that the derivation is correct. We draw conclusions that the electric voltage induced by the pressure-driven flow through complex microchannels almost shows linear dependence on the porosity, which is inversely proportional to the dimensionless electroosmotic radius k . Moreover, the dimensionless parameter δ influences the electric voltage slightly. Hence, the theoretical model can be directly used in engineering applications and guides scientific practice.

Acknowledgments

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References

- [1] Löbbus M, Sonnfeld J, Van Leeuwen H, et al. An improved method for calculating zeta-potentials from measurements of the electrokinetic sonic amplitude. *J. Colloid Interface Sci.* 2000; 229: 174-183.
- [2] Erickson D, Li D, Werner C. An improved method of determining the ζ -potential and surface conductance. *J. Colloid Interface Sci.* 2000; 232: 186-197.
- [3] Chen C. Fully-developed thermal transport in combined electroosmotic and pressure driven flow of power-law fluids in microchannels. *Int. J. Heat Mass Transfer.* 2012; 55: 2173-2183.
- [4] Cho C, Chen C, Chen C. Electrokinetically-driven non-Newtonian fluid flow in rough microchannel with complex-wavy surface. *J. Non-Newtonian Fluid Mech.* 2012; 173-14: 13-20.
- [5] Alkan M, Karadaş M, Doğan M, et al. Zeta potentials of perlite samples in various electrolyte and surfactant media. *Colloids Surf. A.* 2005; 259: 155-166.
- [6] Zhu Q, Xie M, Yang J, et al. Analytical determination of permeability of porous fibrous media with consideration of electrokinetic phenomena. *Int. J. Heat Mass Transfer.* 2012; 55: 1716-1723.
- [7] Zhao C, Yang C. Advances in electrokinetics and their applications in micro/nano fluidics. *MI-CROFLUID NANOFLUID.* 2012; 13: 179-203.
- [8] Pelley A, Tufenkji N. Effect of particle size and natural organic matter on the migration of nano- and microscale latex particles in saturated porous media. *J. Colloid Interface Sci.* 2008; 321: 74-83.
- [9] Liu Y, Yu B, Xiao B. A fractal model for relative permeability of unsaturated porous media with capillary pressure effect. *FRACTALS.* 2007; 15: 217-222.
- [10] Li Y, Zhu Q. A model of coupled liquid moisture and heat transfer in porous textiles with consideration of gravity. *NUMER HEAT TR A-APPL.* 2003; 43: 501-523.
- [11] Van der Heyden F, Bonthuis D, Stein D, et al. Power generation by pressure-driven transport of ions in nanofluidic channels. *NANO LETT.* 2007; 7: 1022-1025.
- [12] Zhu Q, Li Y. Numerical simulation of the transient heat and liquid moisture transfer through porous textiles with consideration of electric double layer. *Int. J. Heat Mass Transfer.* 2010: 1417-1425.
- [13] Yu B, Cai J, Zou M. On the physical properties of apparent two-phase fractal porous media. *VADOSE ZONE J.* 2009; 8: 177-186.
- [14] Yang C, Li D, Masliyah J. Modeling forced liquid convection in rectangular microchannels with electrokinetic effects. *Int. J. Heat Mass Transfer.* 1998; 41: 4229-4249.
- [15] Zhu Q, Xie M, Yang J, et al. A fractal model for the coupled heat and mass transfer in porous fibrous media. *Int. J. Heat Mass Transfer.* 2011; 54: 1400-1409.
- [16] Yu B, Cheng P. A fractal permeability model for bi-dispersed porous media. *Int. J. Heat Mass Transfer.* 2002; 45: 2983-2993.
- [17] Van der Weg P. The electrochemical potential and ionic activity coefficients. A possible correction for Debye-Huckel and Maxwell-Boltzmann equations for dilute electrolyte equilibria. *J. Colloid Interface Sci.* 2009; 339: 542-544.