

# Experimental Analysis and Modeling Research of the Morphological Structure of the Weft-Knitted Loop

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## Abstract

To address the limitations of the traditional idealized loop model, this study conducted experimental analysis and mathematical model for the morphological structure of the weft-knitted loop, and research on the method for describing the weft loop's structure and parameters. Because the features of the loop's structure is consistent with the Non-uniform Rational Basis Spline (NURBS), three degree NURBS curve was used to establish the model of the weft loop, and the corresponding value points' coordinate can be obtained by the experimental analysis of the effect of the fabric structure from the structure parameters. In addition, based on the features of the different weft-knitted structure, the models of plain and pattern stitch have been established. Research shows NURBS curve modeling can be applied to a more complex weft tissue as well as the basic and variation tissue.

*Keywords:* Weft Loop's Structure; Morphological Structure; Non-uniform Rational Basis Spline; Experimental Analysis; Loop Structure's Modeling

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## 1 Introduction

There are many different varieties and patterns of the weft-knitted fabric, and weft CAD technology can greatly improve the design and the production efficiency of the weft-knitted fabric. However, due to the complexity of the weft-knitted structure, it is difficult to use mathematics to accurately describe the loop model [1]. As a result, further development of the weft CAD technology has been constrained. Limitations of the traditional models can no longer meet the requirements of the modern weft CAD technology. Therefore, it is important to establish the corresponding loop model, and at the same time realize the simulation of the fabric structure. Based on the analysis of the morphological variation of the weft-knitted loop structure, this study uses NURBS' curve to establish a model for the weft knitted structure.

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## 2 The General Mathematical Description for Ideal Loop Model of Weft Knitting

The complexity of the morphological structure of the weft is caused by many factors. The unit of the knitted structure is the loop. Its morphological structure is created by the variation at which the yarn is space bended [2]. When each section of the loop is under a force, the yarn moves which leads to the instability of the morphological structure of the weft-knitted fabric.

In the traditional research on the weft-knitted structure, hypotheses were made to reduce the complexity of the mathematical description. Based on characteristics of the loop structure of the weft-knitting, the ideal loop model can be established as shown in Fig. 1. The unit loop's structure consists of a head and two side limbs or legs. The main structure parameters include the width of loop  $W$ , the height of loop  $H$ , the height of the leg loop  $h$ , the length of the leg loop  $l$ , the whole length of loop  $L$  ( $L_1$  is the length without considering the separation distance,  $L_2$  is the length with considering the separation distance). In the ideal loop model, the head loop and the sinker loop are considered as circular, the leg loop is considered as linear, and the yarn a cylinder in which its diameter is labeled as  $d$ . The relation between the loop structure parameters can be calculated [3]:

$$\begin{aligned}
 W &= 4d & h &= \sqrt{(4d)^2 - (2d)^2} = 3.464d \\
 l &= \sqrt{h^2 + d^2} = \sqrt{13}d = 3.606d \\
 H &= h + 4d = 7.464d \\
 \begin{cases} L_1 = 2 \times 1.5d\pi + 2 \times 3.606d \approx 16.64d \\ L_2 = 2p + w + 5.94d \end{cases} & & & (1)
 \end{aligned}$$

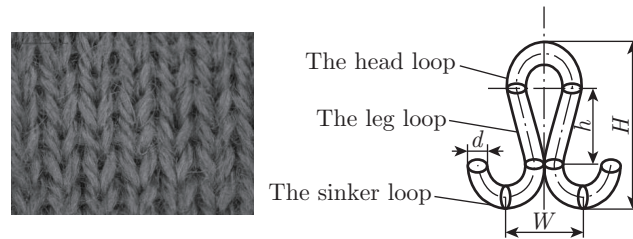


Fig. 1: The structure of plain and the ideal loop model

Therefore, the size performance of the weft-knitting depends on the length of the loop in its relaxed state. The morphological features of the different fabrics depend on the fabric structure and the length of the loop.

## 3 Experimental Analysis of the Actual Loop Morphological Structure

Based on the analysis of the loop morphological structure, to establish a loop model consistent with reality, the limitations of the traditional loop model must be addressed. The limitations include: the model curve is discontinuous and there is a mismatch with the actual yarn's structure;

the simplicity of the model makes it difficult to control freely and describe the loop morphological structure correctly.

NURBS loop modeling is a method to interpolate three or more points on a curve to create a smooth one, and the actual morphological structure is directed by the control points [4]. In the weft-knitted structure, the yarn is a continuous entirety, so the yarn curve modeling requires the use of quadratic equation, in which its features are consistent with the three-degree non-uniform rational basis spline. Here, non-uniform indicates that each control point can be controlled separately, and the rational indicates that the curve or the surface is defined by a quadratic polynomial [5]. Therefore, using the NURBS to establish a model for the weft-knitted loop's structure is appropriate.

### 3.1 A Method of Experimental Analysis of the Loop Structure

Using the NURBS we can establish a model for the weft-knitted loop's structure, and also control the variation of space morphological structure by setting the value points and confirming the power factors. The formula of NURBS curves is [6]:

$$Q(u) = \frac{\sum_{i=0}^n W_i P_i N_{i,k}(u)}{\sum_{i=0}^n W_i N_{i,k}(u)} \quad (0 \leq u \leq 1, k = 3) \quad (2)$$

$$\begin{cases} N_{i,0}(u) \begin{cases} 1(u_i < u < u_{i+1}) \\ 0 \end{cases} \\ N_{i,k}(u) = \frac{u - u_i}{u_{i+k} - u_i} N_{i,k-1}(u) + \frac{u_{i+k+1} - u}{u_{i+k+1} - u_{i+1}} N_{i+1,k-1}(u) \end{cases} \quad (3)$$

Here:  $P_i$  is the location vector of the control points;  $W_i$  is the correlation power factor of  $P_i$ ;  $N_{i,3}(u)$  is the  $k$  degree B-Spline basic function.

Based on the characteristics of the NURBS curve, the model of the weft-knitted loop can be controlled freely, which overcomes the limitations of the traditional loop model.

### 3.2 Orthogonal Experimental Analysis of the Loop Structure

The structure's parameters of the weft loops are: the head loop's width  $A$ , the sinker loop's width  $B$ , loop's height  $H$ , the fabric's thickness  $T$ , the minimum distance of the loop's ring column  $l$ , the yarn's diameter  $d$ . These parameters are closely related to the loop's structure, and each of them affects the loop's structure differently. The orthogonal experiment can be used to analyze the degree of impact on the length of loop from the parameters, and research the relation between the loop's morphological structure and its parameters [7], which can be used to verify the accuracy of the NURBS modeling for the loop's structure. The level of each factor is shown in Table 1.

Based on Table 1, the orthogonal table  $L_8(2^7)$  is chosen and the fabrics with the same structure are knitted to do the test. The experimental data is shown in Table 2.

In Table 2,  $K_i$  indicates the sum of the loop's length of the level  $i(i=1, 2)$  in each list, range  $R = |K_1 - K_2|$ , sum of square  $S = \frac{1}{n} \sum_i (K_i - K_{i-1})^2$ , ( $n$  is time of the test,  $n = 8, i = 2$ ).

Table 1: The level of each factor

Level	Factor					
	$T$ (mm)	$B$ (mm)	$A$ (mm)	$H$ (mm)	$l$ (mm)	$d$ (mm)
1	3.80	4.40	4.48	4.90	0.74	1.12
2	3.60	4.95	5.06	3.27	0.77	0.98

Table 2: The data of experiment

Factor	List							Loop's length (mm)
	1	2	3	4	5	6	7	
	$l$	$T$	$B$	$A$	—	$H$	$d$	
1	1	1	1	1	1	1	1	11.23
2	1	1	1	2	2	2	2	10.92
3	1	2	2	1	1	2	2	11.07
4	1	2	2	2	2	1	1	12.15
5	2	1	2	1	2	1	2	11.70
6	2	1	2	2	1	2	1	11.34
7	2	2	1	1	2	2	1	10.75
8	2	2	1	2	1	1	2	12.06
$K_1$	45.37	45.19	44.96	44.75	45.70	47.14	45.47	
$K_2$	45.85	46.03	46.26	46.47	45.52	44.08	45.75	
Range $R$	0.48	0.84	1.30	1.72	0.18	3.06	0.28	
Sum of square $S$	0.0288	0.0882	0.2112	0.3698	0.0040	1.1700	0.0098	

Table 2 shows the parameters are closely related to the length of loop. The results of the significance analysis are shown in Table 3.

In the Table 3, the degree of freedom of each factor  $f$ =level-1, the sum degree of freedom  $f = n - 1$  ( $n$  is time of the test,  $n=8$ ,  $i=2$ ), mean square  $V = S/f$ , the significance value  $F = V/V_e$ .

Table 3: The significance analysis of each parameter

Variance	Sum of square $S$	Degree of freedom $f$	Mean square $V$	Value $F$
$l$	0.0288	1	0.0288	7.2
$T$	0.0882	1	0.0882	22.05
$B$	0.2112	1	0.2112	52.80
$A$	0.3698	1	0.3698	92.45
$H$	1.1700	1	1.1700	292.5
$d$	0.0098	1	0.0098	2.45
Error $e$	0.0040	1	0.0040	
sum	1.8818	7		

From identifying that  $F_{0.1}(1, 1)=39.86$ ,  $F_{0.05}(1, 1)=161.4$ ,  $F_{0.025}(1, 1)=647.8$ , we can conclude the loop's height is the most significant, followed by the head loop's width, and the sinker loop's width. The order of the structure parameter's significance is  $H > A > B > T > l > d$ . The orthogonal experiment reflects that the loop's length is determined by the structure parameters of the weft loop (the head loop's width  $A$ , the sinker loop's width  $B$ , loop's height  $H$ , the fabric's thickness  $T$ , the minimum distance of the loop's ring column  $l$ , the yarn's diameter  $d$ ). Therefore, it is correct to establish the model for the loop's structure using NURBS curve by setting the value points based on the structure parameters.

## 4 Research of Modeling for Weft Loop Structure Based on NURBS

In the weft loop structure, assuming that the yarn is a continuous entirety, three-degree non-uniform rational basis spline modeling can be used to reflect the morphological nonlinearity of the actual fabric. The NURBS loop model can describe the variation of the space morphological structure by setting the value points and confirming the power factors.

The fabric's basis morphological structure and the trivial status of the fabric's surface, such as the loop's light, the texture, the yarn's bending deformation, are observed by the fabric's Scanning Electronic Microscope (SEM) [9].

Based on the loop's scanning electronic microscope as shown in Fig. 2, the weft-knitted loop structure's model can be established by NURBS curve. The unit loop model is shown in Fig. 3, which is controlled by nine value points [10].

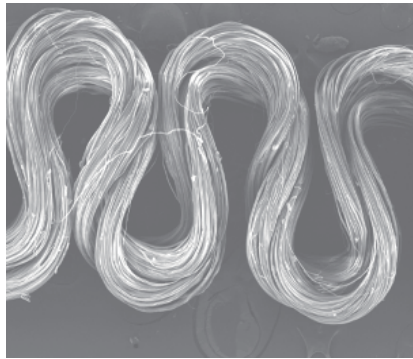


Fig. 2: The scanning electronic microscope of the fabric and the loop

According to the unit loop model, a NURBS model of plain loop's structure can be established by combining the actual plain's morphological structure as shown in Fig. 3. The value points,  $Q_{(1)}$ ,  $Q_{(3)}$ ,  $Q_{(5)}$  and  $Q_{(7)}$ , are the connecting points between the loops of the plain's NURBS model.

The results from the experiments reflect that the value points of the plain's NURBS model and the relation between the loops depend on the main parameters of the loop's structure. The parameters of the weft loop are the head loop's width  $A$ , the sinker loop's width  $B$ , loop's height  $H$ , the fabric's thickness  $T$ , the minimum distance of the loop's ring column  $l$ , the yarn's diameter  $d$ , a single weft thread's height  $C$ , the vertical column of loop's width  $W$  and so on. The value

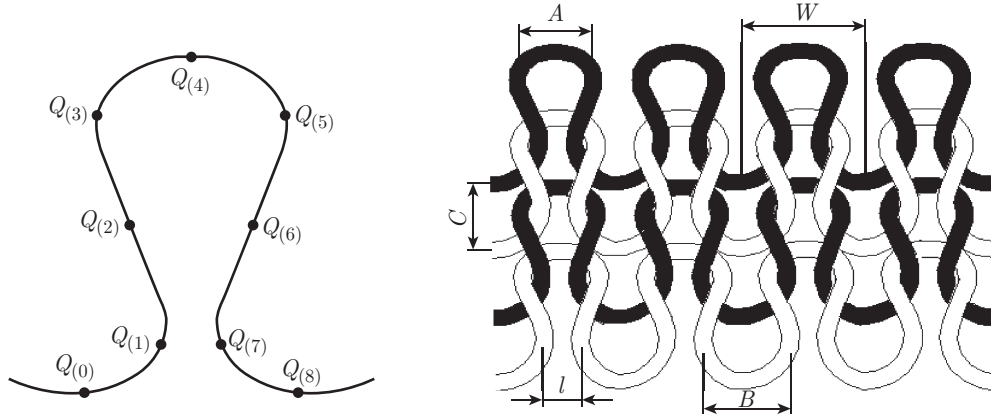


Fig. 3: The unit loop model and the plain loop's model

points' coordinates of the plain's NURBS model can be calculated based on the parameters of the weft loop. The relation between the value points  $Q_{(i)}$  and the parameters can be obtained.

$$\begin{aligned} A &= |x_{Q(5)} - x_{Q(3)}|, & B &= 2|x_{Q(1)} - x_{Q(0)}|, & H &= |y_{Q(4)} - y_{Q(0)}|, \\ T &= |z_{Q(1)} - z_{Q(0)}|, & l &= |x_{Q(7)} - x_{Q(1)}| \end{aligned} \quad (4)$$

Through the transformation calculation, the value points' coordinates of the unit NURBS model can be calculated:

$$\begin{aligned} Q_{(0)}(0, 0, 0), & \quad Q_{(1)}\left(\frac{B}{2}, \frac{H}{4}, T\right), \quad Q_{(2)}\left(\frac{B}{4}, \frac{H}{2}, \frac{T}{2}\right), \quad Q_{(3)}\left(0, \frac{3}{4}H, T\right), \quad Q_{(4)}\left(\frac{B+L}{2}, H, \frac{T}{2}\right), \\ Q_{(5)}\left(A, \frac{3}{4}H, T\right), & \quad Q_{(6)}\left(\frac{3}{4}B + L, \frac{H}{2}, \frac{T}{2}\right), \quad Q_{(7)}\left(\frac{B}{2} + L, \frac{H}{4}, T\right), \quad Q_{(8)}(B + L, 0, 0) \end{aligned}$$

After the plain's NURBS model has been established, the starting position of the fabric's model is set as  $(0, 0, 0)$ , so the coordinates of the unit model in weft  $m$  direction and wale direction can be calculated:

$$Q_{(i)(m,n)}(x, y, z) = \begin{bmatrix} x_i & y_i & z_i \end{bmatrix} \begin{bmatrix} (n-1)W + 1 & 0 & 0 \\ 0 & (m-1)C + 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

NURBS modeling of the curve's morphological structure is controlled by setting the value points, so the loop's structure can be controlled freely [11]. Stitch transferring is an important way of creating sophisticated weft fabrics. Based on the morphological structure and the knitting process of the 3+3 pattern stitch as shown in Fig. 4, the 1+1 pattern stitch's NURBS model has been established as shown in Fig. 5.

The pattern stitch is created by the loops' transferring based on the basis knitting fabric. Relation of the structure parameters has changed accordingly. According to the characteristics of the NURBS repeated nodes, the value points of the pattern stitch's NURBS model can be calculated. If the pattern is created by the loop of the weft  $m$  direction and wale  $n+1$  direction transferring with the loop of the weft  $m$  direction and wale  $n+1$  direction based on the plain



Fig. 4: The actual 3+3 pattern stitch

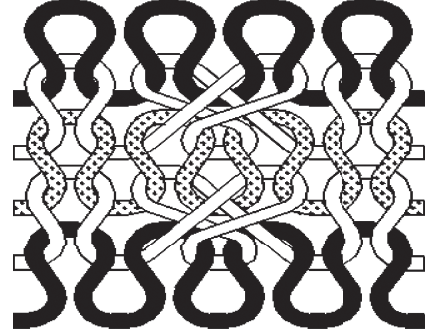


Fig. 5: The 1+1 pattern stitch's NURBS model

fabric, then, the coordinates of the 1+1 pattern stitch's NURBS model in weft  $m$  direction and wale  $n$  direction can be calculated:

$$\begin{aligned}
 Q_{m,n(0)}(x, y, z) &= Q_{m,n-1(8)}(x, y, z) = \left\{ (n-2)W + (B+l), (m-1)C, \frac{T}{3} \right\} \\
 Q_{m,n(1)}(x, y, z) &= Q_{m-1,n(4)}(x, y, z) = \left\{ (n-1)W + \frac{B+l}{2}, (m-2)C + H, \frac{2T}{3} \right\} \\
 Q_{m,n(2)}(x, y, z) &= \left\{ Q_{m+1,n(7)}(x) + \frac{B}{2}, Q_{m,n-1(6)}(y), \frac{T}{3} \right\} = \left\{ (n-1)W + (B+l), (m-1)C + \frac{H}{2}, \frac{T}{3} \right\} \\
 Q_{m,n(3)}(x, y, z) &= Q_{m+1,n+1(1)} = \left\{ nW + \frac{B}{2}, mC + \frac{H}{4}, T \right\} \\
 Q_{m,n(4)}(x, y, z) &= Q_{m,n+1(4)}(x, y, z) = \left\{ nW + \frac{B+l}{2}, (m-1)C + H, T \right\} \\
 Q_{m,n(5)}(x, y, z) &= Q_{m+1,n+1(7)}(x, y, z) = \left\{ nW + (\frac{B}{2} + l), mC + \frac{H}{4}, T \right\} \\
 Q_{m,n(6)}(x, y, z) &= \left\{ Q_{m-1,n+1(3)}(x), Q_{m,n(2)}(y), \frac{T}{3} \right\} = \left\{ nW, (m-1)C + \frac{H}{2}, \frac{T}{3} \right\} \\
 Q_{m,n(7)}(x, y, z) &= Q_{m-1,n(5)}(x, y, z) = \left\{ (n-1)W + A, (m-2)C + \frac{3H}{4}, \frac{2T}{3} \right\} \\
 Q_{m,n(8)}(x, y, z) &= Q_{m,n+1(0)}(x, y, z) = \left\{ nW, (m-1)C, \frac{T}{3} \right\}
 \end{aligned} \tag{6}$$

The morphological structure of the weft loop can be controlled by NURBS curve modeling. The method can also apply to the complex fabric's modeling, which solves the limitations of traditional ideal loop model.

## 5 Simulation for the Weft-knitting

The 3D image processing technology OpenGL has many useful functions, and its GLU function provides an interface to create NURBS curve by setting the control points, the node vectors, and the power points and so on [12]. Based on the theoretical research, the 3D simulation for the weft-knitting can be gained (as shown in Fig. 7). In OpenGL, the solving process of creating NURBS curve is shown in Fig. 6.

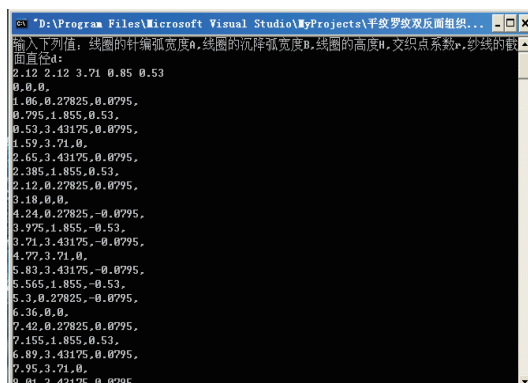


Fig. 6: The solving process in OpenGL

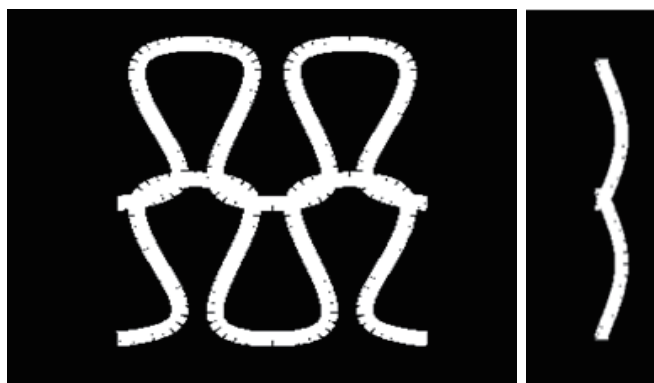


Fig. 7: The 3D simulation for the fabric

Here, based on modeling the 3D NURBS loop structure, the 3D simulation for the weft-knitting can be achieved by setting the structure parameters. The result of the simulation can describe the fabric's morphological structure, which reflects the correctness of NURBS modeling.

## 6 Conclusions

(1) Based on the morphological structure of weft loop and the variation regularity of fabric, a mathematic description method of the weft loop's structure and parameters was researched for morphological structure of weft loop to address the limitations of traditional ideal loop model.

(2) The model curve of the loop's structure is connected smoothly by using the three-degree non-uniform rational basis spline to establish the weft loop's model. The characteristics of loop structure are consistent with the NURBS curve, so the weft loop's NURBS model can be controlled freely, which reflects the variation of the loop's space morphological structure correctly.

(3) Analysis of the degree of influence to the length of loop from the parameters based on an orthogonal experiment and the internal relations between the variation of the loop's morphological structure and the parameters of the fabric can be verified. The results of the research reflect the loop's morphological structure, which can be described correctly by using the NURBS to establish the model and solve the value points.

(4) The models of plain, rib fabric and pattern stitch have been established by three-degree NURBS, which demonstrates that the method of NURBS modeling not only applies to weft basis and variation fabrics, but also to complex weft fabrics.

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